THEORY OF COMPUTATION

FINITE AUTOMATA

SOLUTIONS

1. Finite automata can be used in

(a) Lexical Analysis
(b) Syntax Analysis
(c) Semantic Analysis
(d) None of these

Solution: Option (a)

Explanation:

Finite Automata is used during Lexical Analysis to recognize tokens.

2. Consider the following statements.

(i) Kleen closure of an empty language is non-empty.
(ii) Some infinite languages are regular.
(iii) L = $\{a^P / P$ is a prime number$\}$ is a regular language.

(a) Only (i) is true
(b) Only (ii) & (iii) are true
(c) Only (i) & (ii) are true
(d) All are true

Solution: Option (c)

Explanation:

(i) Kleen closure of an empty language has the only string $\in$. So, it is non-empty.

$\phi^* = \{\in\}$

(ii) Some infinite languages are regular

Eg: L = $\{a^n / n \geq 0\}$ is regular

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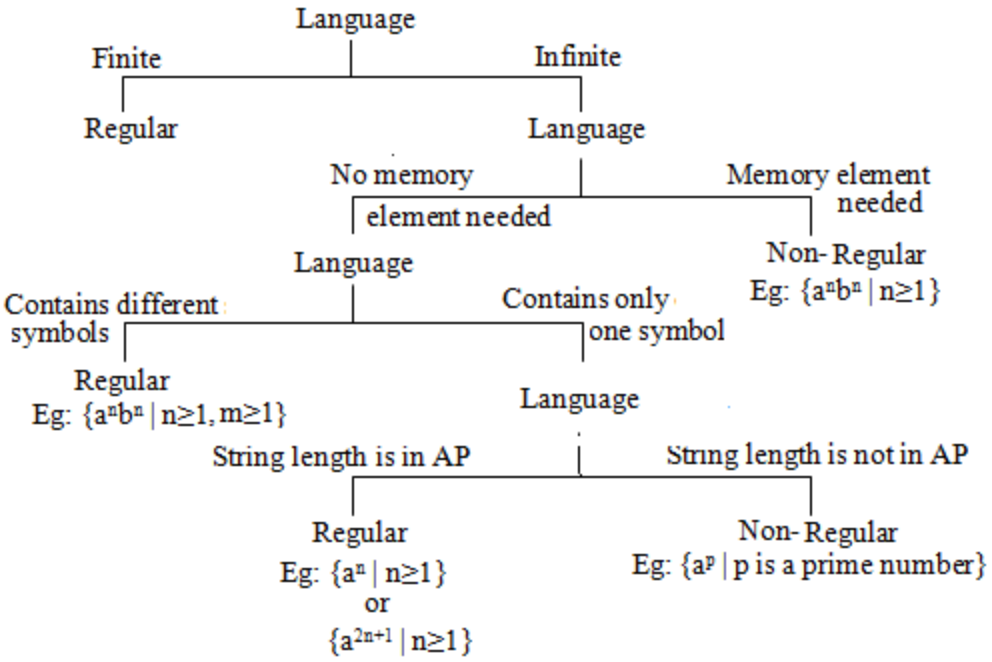

∴ (iii) is not true.

3. For a given NFA with n state, the maximum number of state possible in the equivalent DFA is

(a) 1 (b) n
(c) n + 1 (d) 2^n

Solution: Option (d)

Explanation:

Max. No. of states in DFA is 2^n
& Min. No. of states is 1 for an* NFA with n states.

4. The minimal finite automata that accepts all strings of a's & b's, where the number of a's is atleast 'n' contains _____ number of states.

(a) n – 1 (b) n
(c) n + 1 (d) n + 2

Solution: Option (c)

Explanation:

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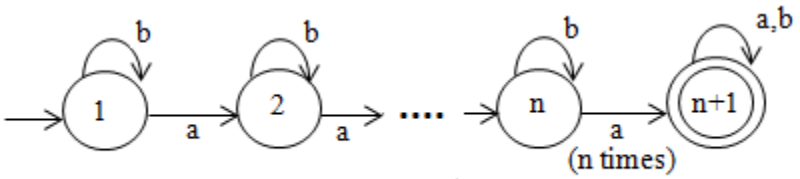
Given Σ = {a, b}
|w| ≥ n, where w ∈ {a, b}*

The minimal automata accepting this language has n + 1 states.



5. Consider the minimal Finite automata that accepts all the strings of a's & b's where each string contains

(i) exactly 5 a's
(ii) atmost 5 a's

The No. of states in each case respectively are:

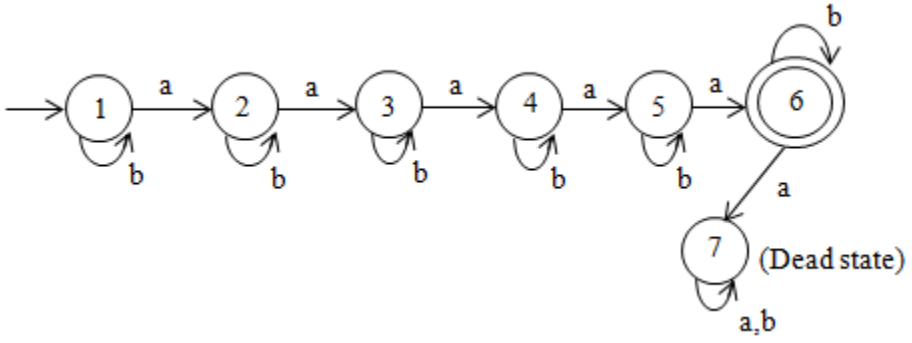
(a) 6, 6
(b) 7, 7
(c) 6, 7
(d) 7, 6

Solution: Option (b)

Explanation:

Given Σ = {a, b}
(i) $|w|_a = 5$, where w ∈ {a, b}*

The minimal finite automata accepting exactly 5 a's has 7 states (including a dead state).



(ii) $|w|_a \leq 5$, where w ∈ {a, b}*
The minimal finite automata accepting atmost 5 a's has 7 states (including a dead state).

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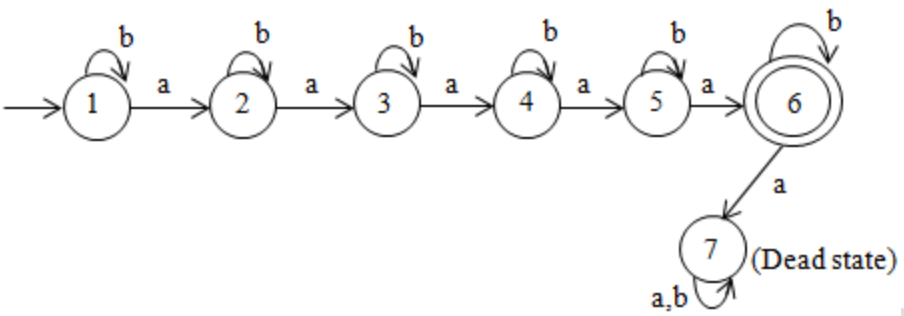


Note: $|w|_a = n \Rightarrow n+2$ states
$|w|_a \leq n \Rightarrow n+2$ states
$|w|_a \geq n \Rightarrow n+1$ states

6. Consider the Minimal Finite Automata that accept all the strings of a's & b's where no. of a's is congruent to 2 mod 3. The no. of states in this automata is

(a) 4 (b) 3
(c) 2 (d) 1

Solution: Option (b)

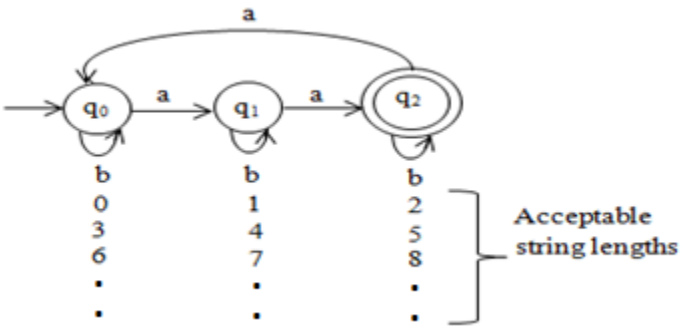
Explanation:

$\Sigma = \{a, b\}$
$w \in \{a, b\}^*, |w|a \equiv 2(\text{mod } 3)$

It means, the automata accepts the string having the no. of a's as 2, 5, 8, 11, 14,……

Clearly, there are 3 states only because of mod 3 and 3rd state will be the final state because of 2.

∴ MFA is



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Note: The no. of states in MFA that accept all the strings of a's and b's where $|w|_a \equiv r(\text{mod} n)$ or $|w|_b \equiv r(\text{mod } n)$ is exactly n, and q_r is final state.

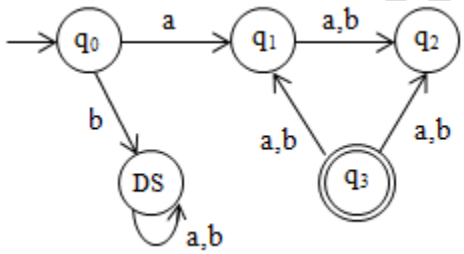
7. What is the number of states in a minimal finite automata that accepts all the strings of a's & b's where each string starts with 'a' and total length of the string is divisible by 3.

(a) 3 (b) 4
(c) 5 (d) 6

Solution: Option (c)

Explanation:

w = ax & $|w| \equiv 0(\text{mod}3)$



8. What is the number of states in the minimal finite automata that accepts all the strings of a's and b's where each string starts with 'bba' and the length of the string is congruent to 2(mod 6).

(a) 8 (b) 9
(c) 10 (d) 11

Solution: Option (c)

Explanation:

w = bbaX & $|w| \equiv 2(\text{mod } 6)$ ➡ accepts strings of length 8, 14, 20, 26, ……

↓

0, 1, 2, 3, 4, 5

DFA for string starting with bba:

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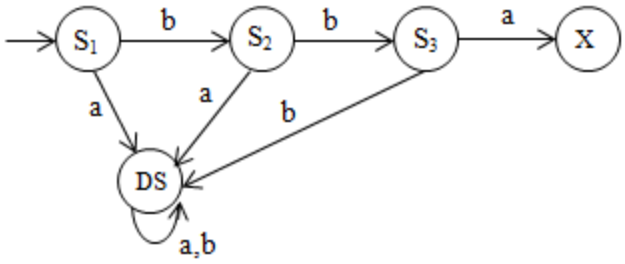
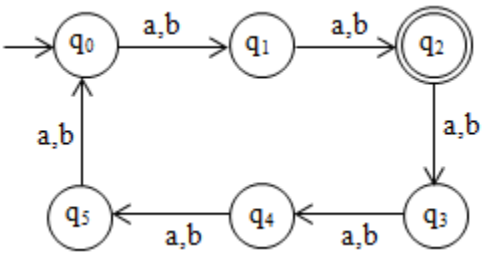
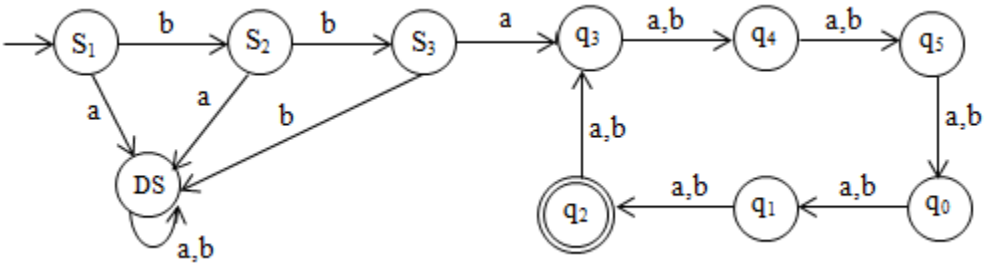


DFA for string length congruent to 2(mod6):



Now, the minimal finite automata is



Note: The minimal finite automata that accept all the strings of a's & b's where every string starts with substring 'S' of length m and the length of the string is congruent to r(mod n) contains exactly m+1+n states.

$\Sigma = \{a, b\}$
$w = Sx \;\&\; |w| \equiv r(\text{mod } n)$
$\downarrow$
$|S| = m$
No. of states = m+n + 1

9. Construct the minimal finite automata that accept all the strings of 0's & 1's is where the integer equivalent is congruent to 1(mod 4). What is the no. of states in minimal finite automata?

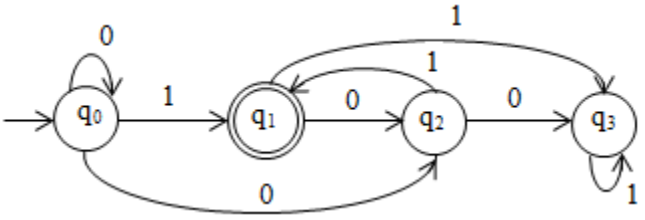
(a) 4 (b) 3
(c) 2 (d) 1

Solution: Option (b)

Explanation:

Binary number ≡ 1(mod 4)

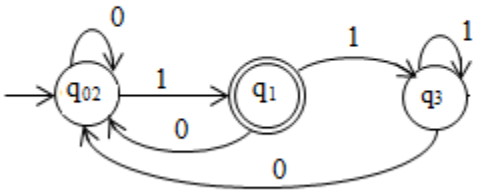
δ	0	1
→q_0	q_0	q_1
*q_1	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_3



We can further minimize the finite automata:

Merge q_0 & q_2

δ	0	1
q_{02}	q_{02}	q_1
q_1	q_{02}	q_3
q_3	q_{02}	q_3



We can also do this by grouping:
{q_0 q_2 q_3} {q_1}
{q_0 q_2} {q_3} {q_1}

10. Construct the minimal finite automata that accept all the strings of 0's and 1's where the integer equivalent of the binary string is congruent to 3 mod 6. What is the number of states in the minimal finite automata?

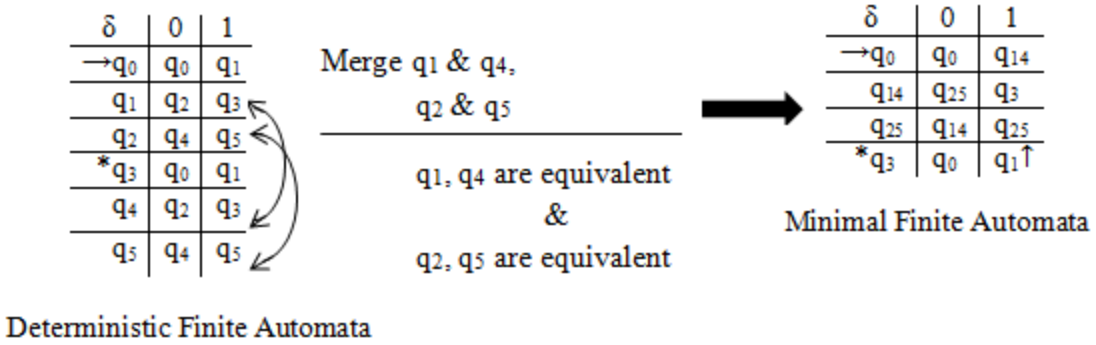
(a) 6 (b) 5
(c) 4 (d) 3

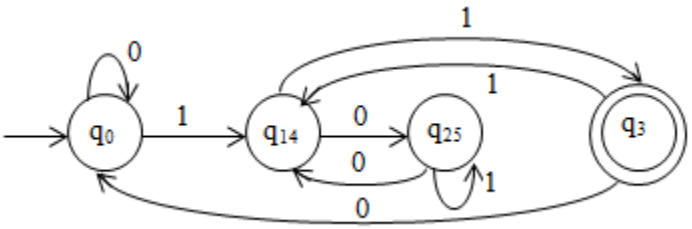
Solution: Option (c)

Explanation:

Given the integer equivalent of binary string is ≡ 3 mod 6.

∴ There are 6 states in DFA.





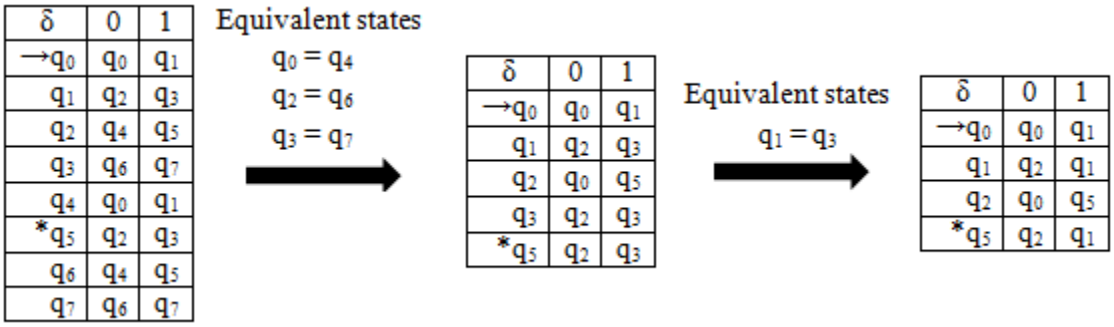
11. Given Σ = {0, 1}, the minimal finite automata that accepts all the strings from the given alphabet where the integer equivalent of the binary string is congruent to 5(mod 8) has _____ no. of states.

(a) 8 (b) 7
(c) 5 (d) 4

Solution: Option (d)

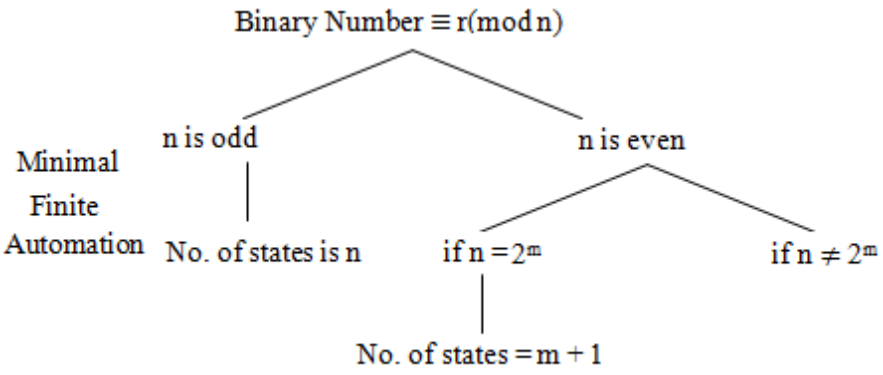
Explanation:

Transition table of DFA:



∴ The minimal finite automata that accepts the integer equivalent of strings formed by 0's & 1's where ≡ 5(mod 8) has 4 states.

Note:



Since it is congruent to 5(mod 8) and $8 = 2^3$, number of states in MFA is 3+1 = 4 states

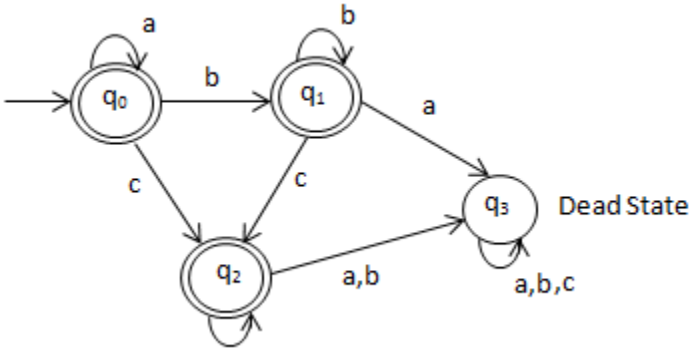
12. Given language L = $\{a^m b^n c^p \mid m,n,p \geq 0\}$, Σ = {a, b, c}. Find the number of states in minimal finite automata of the above language.

(a) 3 (b) 4
(c) 5 (d) 6

Solution: Option (b)

Explanation:

The given language contains
∈, a^m, b^n, c^p, $a^m b^n$ $b^n c^p$, $a^m c^p$, …….



Minimum length of the string = 0+0+0 = 0

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13. Given L = $\{a^m b^n c^p d^q \mid m,n,p,q \geq 0\}$, Σ = {a, b, c, d}, find the number of states in the minimal finite automata of the language.

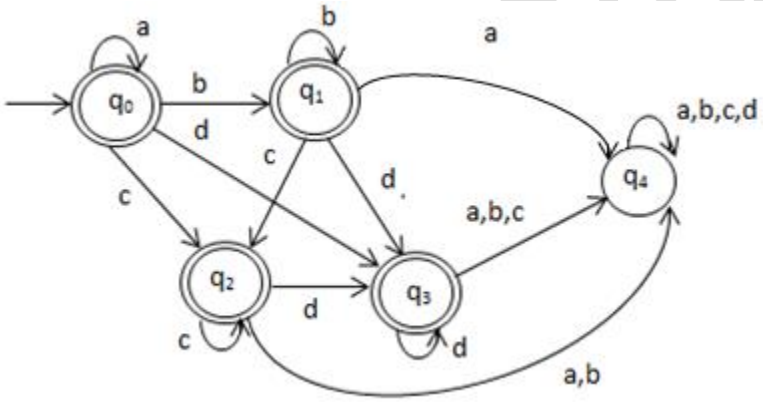
(a) 3 (b) 4
(c) 5 (d) 6

Solution: Option (c)

Explanation:

The given language contains
∈, a^m, b^n, c^p, d^q, $a^m b^n$, $b^n c^p$, $c^p d^q$, $a^m b^n c^p$, …..

Minimum length = 0+0+0+0 = 0



Note: If Given language is of the form

L = $\{a_1{}^{n_1} a_2{}^{n_2} \ldots\ldots a_k{}^{n_k} \mid n_1, n_2, \ldots\ldots, n_k \geq 0\}$
& $\Sigma = \{a_1, a_2, \ldots\ldots, a_k\}$

Then Number of States in minimal finite automata = |Σ| + 1
Eg: Let L = $\{a^{n_1} b^{n_2} \ldots\ldots z^{n_{26}} \mid n_1, n_2, \ldots\ldots, n_{26} \geq 0\}$

No. of states in MFA is |Σ| + 1 = 26 + 1 = 27

14. Given a language L = $\{a^m b^n \mid m \geq 0, n \geq 2\}$, Σ={a, b}. Find the number of states in the minimal finite automata that accepts the above language.

(a) 3 (b) 4
(c) 5 (d) 6

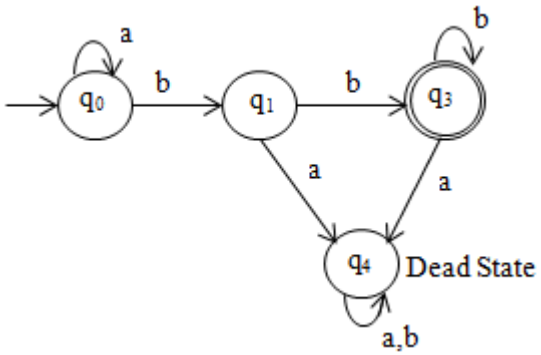
Solution: Option (b)

Explanation:

The language contains
b^2, ab^2, a^2b^2, ab^3, ……

Minimum length = 2

∴ No. states = 2+2 = 4



MFA

15. L = $\{a^m b^n \mid m \geq 0, n \geq 2016\}$, $\Sigma = \{a, b\}$. Find the no. of states in the minimal finite automata that accepts this language.

(a) 2016 (b) 2017
(c) 2018 (d) 2019

Solution: Option (c)

Explanation:

If the given language is of the form
$L = \{a^n b^m \mid m \geq 0, n \geq k\}$

Then no. of states in minimal finite automata that accepts this language is k+2 (k is the minimum length of any string).

∴ Language $L = \{a^m b^n \mid m \geq 0, n \geq 2016\}$

No. of states = 2016 + 2 = 2018

16. The number of states in MFA that accept all the strings of a's & b's where each string contain odd occurrence of the substring ab is

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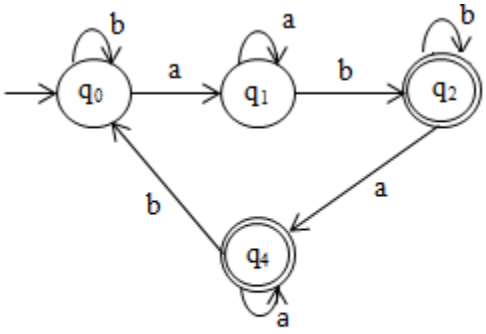
(a) 4 (b) 5
(c) 6 (d) 7

Solution: Option (a)

Explanation:



L= {ab, bbaab, babaa, bababab, ……}

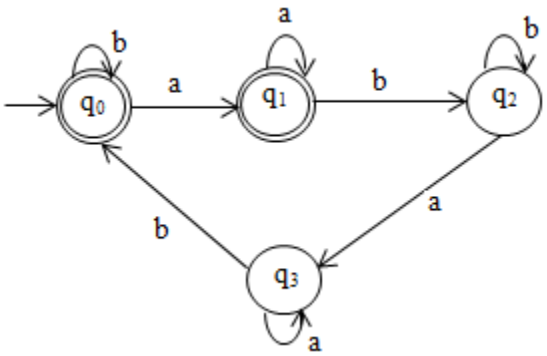
17. The number of states in MFA that accepts all the strings of a's & b's where each string contains even occurrences of substring ab

(a) 4 (b) 5
(c) 6 (d) 7

Solution: Option (a)

Explanation:

L = {∈, abab, babaaab, ……}



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18. What is the number of final states in the minimal finite automata, where Σ = {a, b}, if every string start with "aa" and length of string is not congruent to 0(mod 4)

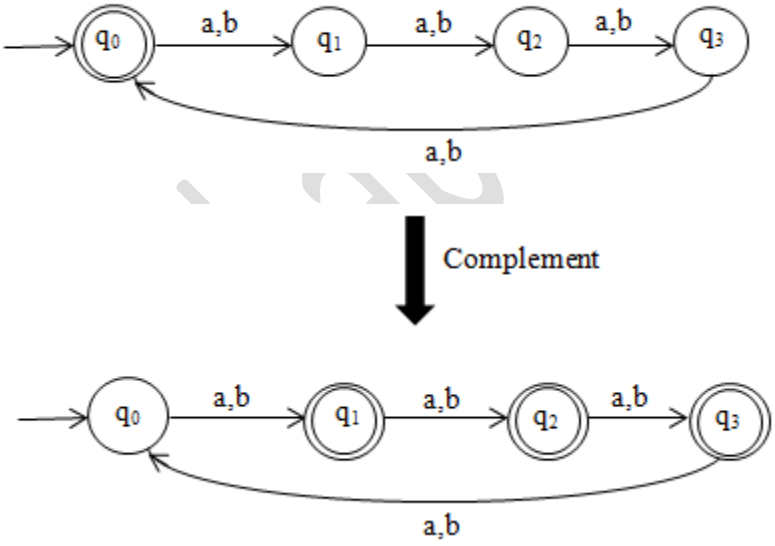
(a) 4 (b) 3
(c) 2 (d) 1

Solution: Option (b)

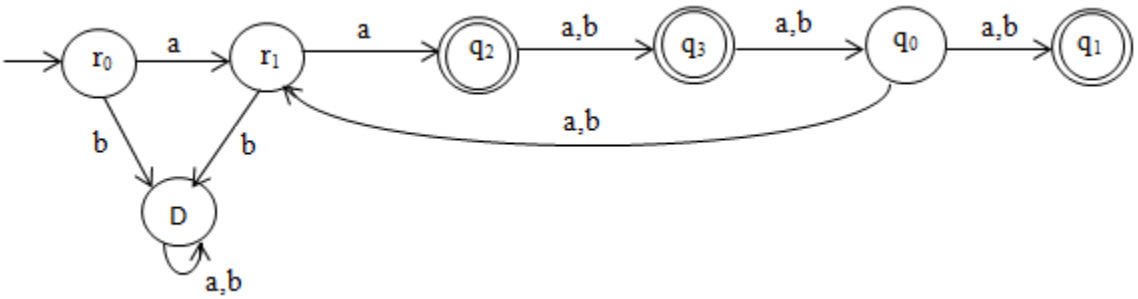
Explanation:

Given that length of string is not congruent to 0(mod a).

Let us construct automata for the language that accepts strings of length congruent to 0(mod 4) and complement it to get the automata that accepts all the strings of lengths not congruent to 0(mod 4).



The minimal finite automata that accepts strings that start with "aa" and length of string is not congruent to 0(mod 4) is:



∴ Total no. of final state is 3.

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